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Estimation of Turbofan Engine Performance Model Accuracy and Confidence Bounds

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Abstract

This paper explores the application of Inference and Bayesian Updating principles as a means to efficiently incorporate probabilistic data into the turbine engine status model matching process. This approach allows efficient estimation of nominal model match parameters from test data and also enables quantification of model accuracy and confidence bounds. The basic concepts are developed in detail and formulated into a status matching approach. This method is then applied to a simple surrogate matching problem using a cantilever beam matching exercise to illustrate the methods in a clear and easy-to-understand way. Typical results are presented and are directly analogous to status matching of a gas turbine engine cycle model.

Introduction

This paper focuses on developing improved methods for matching high-accuracy engine performance models to test data. The chief motivation for this work is the need to make model predictions match test data as closely as possible and to do so in the minimum possible time and cost. The basic engine performance matching problem is complicated by the fact that the test data has inherent uncertainty which must be accounted for in the matching process. Furthermore, the data typically comes from many sources, each with a varying degree of relevance (legacy test data versus new test data, for instance). Appropriate weight must be apportioned to each piece of data according to its relevance to the current engine. Finally, some parameters must be matched with greater precision than others. Challenges are also presented by continued increases in engine complexity (particularly with regard to control laws), and the need to model an ever-increasing number of phenomena that impact engine performance. The confluence of these factors is a clear and present need to develop rigorous, efficient, and methodical approaches to matching model predictions to test data.

While it is important that the data match be as accurate as possible, it is equally important to be able to quantify the accuracy of the match. Engine performance models are used throughout the design process and impact significant decisions regarding the design and specific commitments to customers. A failure to appreciate the accuracy of the performance model can cause customer commitments to be unattainable or, conversely, to be overly conservative. Similarly, the same issues can lead to overly conservative designs or to field problems resulting from optimistic assumptions about the conditions an engine component will experience in service. Thus, a second ingredient of a useful engine performance model is the ability to specify its uncertainty.

A Typical Status Matching Approach

A general feature of status matching is that the problem becomes more complex and the flexibility greater as more data is available to be considered. The easiest case is a production status match where there are many engines, but few parameters to be matched. Another thing that makes the production status match simpler is that all the data is at sea level static conditions.

The goal of a production status match is to simultaneously match thrust at fan speed, core speed at fan speed, specific fuel consumption (SFC) at thrust and exhaust gas temperature (EGT) at fan speed. Although internal pressures and temperatures are available, they are normally single element probes (control sensors) and are not considered in the matching process.

Typically, the first step is to match thrust at fan speed. This is done by varying the fan pumping (air flow at fan speed) until a thrust match is achieved. In this, or any subsequent step, a reasonableness check is performed to make certain that the resulting map makes sense. This might involve an observation that very little change from the previous representation was required to achieve the match or it could involve a review with the appropriate aero designer or other expert(s).

Step two is to match core speed at fan speed. This will usually be done by adjusting the high pressure compressor flow at speed characteristic. If either high pressure or low pressure turbine flow functions have changed (because of a deliberate design decision), they will be included in this part of the match because they affect the speed/speed relationship.

Step 3 is to match the SFC at thrust. There are many handles available to achieve this match (fan, booster, compressor, high- and low-turbine efficiencies and parasitic flows are the prime movers). The status matcher will be watching the EGT at fan speed during this process because core efficiency changes have a much stronger impact on EGT than low-spool system efficiency changes (roughly two-to-one). The decision of how to match the SFC at thrust is likely to be influenced by factors outside the engine test data (for example, what design changes have been introduced that could change component efficiency). The internal temperatures and pressures could be used to help resolve this issue, but this seldom occurs in practice. If one or more development engine tests have been conducted that might shed light on the apportionment of the cycle efficiency, they might be used to help assign the credit or blame.

The matching of SFC may take care of the EGT at fan speed match, but it might not. If it doesn't, the likely choice to match EGT is to adjust the indicated-minus-true (EGT measurement bias) curve to bring the EGT at fan speed into agreement with the test data. EGT is not really independent from fuel flow, and in the test cell, fuel flow is considered to be the more reliable measurement. Hence, if EGT implies a different answer than fuel flow, the fuel flow will be believed and the EGT bias will be adjusted to make peace. GEAE regularly has experiences that lend support to the wisdom of this approach.

The handling of idle (low power) data is somewhat different. At idle, the important matches are thrust at fan speed, core speed at fan speed, and WF/PS3 (fuel flow divided by HP compressor discharge static pressure) at fan speed. This process is comparable to the high power match described above except for the use of WF/PS3 as the target parameter describing cycle efficiency. The only addition would be for ground and approach idle, EGT at N1 and Fuel flow at N1 matches are also important.

As previously indicated, once the production data match is achieved, there is a question of blending it into the previous altitude match. The ground data cannot be used to fully specify what the model should be at altitude. Hence, the modeler must decide how much to let the ground match alter the altitude match and how to implement that decision.

One can typically assume that the parameters which must be matched in any given status matching exercise will consist, at a minimum, of thrust, SFC, exhaust gas temperature (EGT), and core speed. The parameters free to use in the matching process typically consist of scale factors on turbomachinery efficiency and flow rate.

In all cases, the status matching process starts with an estimate of what the model parameters are predicted to be and then modifies them based on test data. An estimate of confidence bounds on initial parameter estimates is usually available, and provides a complete probabilistic description of the initial model parameters. These model parameters are adjusted using test data which itself has some measurement uncertainty associated with it. Thus, status matching ultimately involves updating of a probability distribution using another probability distribution. Viewed in this context, the status matching process just described closely resembles Bayesian Updating of probabilities. Let us therefore explore the parallels between status matching and Bayesian Updating and assess the potential for using Bayesian Updating to facilitate probabilistic status matching.

"Inverse" Probabilities by Inference and Bayesian Updating

The most promising tools available to assist in the development of accurate status decks are Inference and Bayesian Updating.[1] This section gives an intuitive explanation of both concepts and illustrates how they can be applied to cycle deck status matching. Let us begin by considering ordinary probabilities. If we are given a probability distribution on a variable, x, this distribution can be used to predict the probability that a specific value of x will occur in a given observation using elementary probability theory. Now, consider the reverse of this problem wherein we have an observed value of x and want to use this observation to infer something about the underlying distribution on x from whence it came. If many such observations are available, one can build up a history of x and deduce what the underlying distribution looked like. Moreover, we can quantify our confidence in the estimate of the x-distribution using sample statistics.[2]

Bayesian Updating

What about the inverse situation wherein we already have a prior belief (based on past experience, for example) of what we believe the distribution of x is and would now like to modify our beliefs on x based on new data. Bayesian Updating provides a mechanism for doing this—it can be thought of as an "inverse probability" method. This idea is illustrated in Figure 1 where we are given an initial set of distributions on model parameters based on current beliefs and past observations of model parameters (the priors). Now, given a new observation of model parameters (in this example, parameters "X1-X4" are measured for several specimens), exact Bayesian Updating can be used to produce an updated set of distributions that combines the prior beliefs with the new observations.

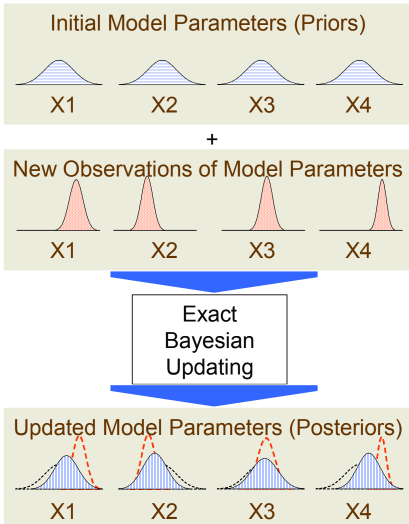

Figure 1: Bayesian Updating of Model Parameters Based on New Observations of Model Parameters.

If the input and output distributions can be assumed to be normal, then simple algebraic formulas can be used to calculate the updated mean and variance of the model parameters.[3] For example, presume that we want to update one of the four parameters shown in Figure 1. The exact Bayesian calculation assumes that the mean (μ_0) and standard deviation (σ_0) of the prior distribution are given. In addition, a mean weighting parameter (κ_n) and a standard deviation weighting parameter (ν_n) are given. These parameters are required to determine the relative weighting of the prior versus the new data point(s) in calculating the value of the posterior. The number of new data points to be used in the Bayesian update, n, also plays a role in determining the relative weight of the prior and new data points. For example, if we had a thousand past observations that were used to estimate the prior distributions and wanted to use Bayesian Updating to add five new data points to the current estimate of the model parameters, κ_n and ν_n would be set at 1,000 and n would be 5. The mean of the updated model parameters can then be computed as:

$$\mu_n = \frac{\kappa_n}{\kappa_n + n} * \mu_n + \frac{n}{\kappa_n + n}\bar{y} \qquad (1)$$

where $\bar{y}$ is the average of the new data points used in the update. The standard deviation of the updated model parameter is computed as:

$$\sigma_n = \sqrt{\frac{\nu_n \sigma_n^2 + (n-1)s^2 + \frac{\kappa_n n}{\kappa_n + n}(\bar{y} - \mu_n)^2}{\nu_n}} \qquad (2)$$

where s is the sample standard deviation of the new data points used in the update. After each update, the new estimates for posterior mean and standard deviation then become the priors for the next update while the model weighting parameters are updated to reflect the number of data points contained in their history:

$$\kappa_n = \kappa_0 + n$$

$$\nu_n = \nu_0 + n$$

Thus, if we had priors containing 1,000 data points and updated the priors with 5 new data points, the new values of κ_n and ν_n would be 1,005. Equation (1) is effectively the weighted average of the means of the prior and additional data points, with the weighting being provided by the number of data points in each set. Note that equation (2) contains three terms, one for the contribution of the prior standard deviation, one for the contribution of the sample deviation, and a third to account for the spread of the sample mean from the prior mean.

These equations are intuitively simple and are quite easy to use. Note, however, that Eqs. (1) and (2) are only valid when the prior and posterior distributions are normal. They do not hold for the general case having any shape probability distribution. This assumption may seem somewhat restrictive, the majority of updating problems can be assumed to be sufficiently close to normal to yield useful results. Finally, Bayesian purists should note that Eqs. (1) and (2) represent only the maximum likelihood estimate for the updated distribution parameters and do not represent the distribution of possible updated model parameters themselves.

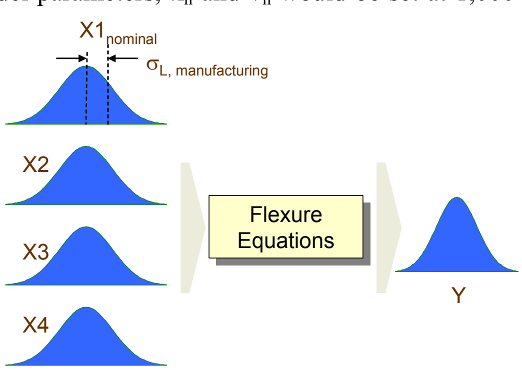

Figure 2: The "Standard" Probability Problem: Given Distributions on Inputs, Determine the Distribution on the Output.

Inference

A variation of this "inverse probability" problem that is very germane to this work is the case where the distribution on x is related to some output parameter, Y=f(X). In this case, instead of being given an updated value of X, *we are given an updated value of Y and must use this to infer something about the underlying distributions on X that gave rise to that observed value of Y.* We will refer to this herein as "Inferenceing" as distinguished from "Bayesian Updating" discussed previously. To understand how to handle this situation, consider the "standard" probability problem involving a function Y=f(X) shown in Figure 2. In the "standard" probability problem, we are given estimated distributions on inputs and can then use a variety of techniques (Monte Carlo Simulation, Fast Probability Integration, etc.) to determine the distribution on output parameters.

The "inverse" probability problem solved by Inferenceing is shown in Figure 3. In this case, we already assumed input distributions (known as *prior distributions*) which have been used to find the output distribution on Y. We are now given an observed value of Y and desire to extract that portion of the input distributions that corresponds to the observed value of Y. That portion of the input distributions are depicted as smaller light-colored distributions inside the original distributions at the right side of Figure 3, and are known as the *posterior distributions*. The posteriors will generally have a different mean and standard deviation than their parent (prior) distributions. The change from the prior is due to the influence of the new observation of Y translated into equivalent distributions on the input parameters. It should be noted that although the posteriors are depicted in Figure 3 as being a subset of the original prior distributions, the total area under the distribution curve must still be equal to 1.0. Therefore the small distributions should actually be scaled up to have the same area as the original distributions.

"Maximum Entropy" Inference

It should be evident by now that Inference is a key element for enabling a Bayesian approach to status matching. However, nothing has been said up to this point regarding how to select the initial distributions used in the Inference process. One may at first be tempted to use the current estimates of model parameters themselves as the distributions used in the Inference process, but this is misleading. What we want is for each new measurement of Y to tell us something about the underlying X's *independent* of what we think the X's should be. If we use current estimate of the X-distributions, it introduces *bias* into Inference process based on expectations of what we think the solution should be.

If the inferred X-distributions are to be a reflection of the observed Y only, one must use a *completely unbiased* prior distribution. The only unbiased prior distribution is a uniform distribution—it says nothing about what the expected values of X's are except that they must lie within a prescribed range. This is a well-known approach used in the literature, and is sometimes referred to as the "maximum entropy" distribution because it contains the maximum amount of Shannon entropy from an information theoretic point of view.[4]

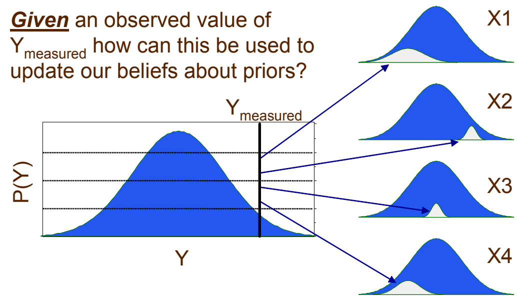

Figure 3: "Inverse" Probability Through Inference.

This idea is illustrated in Figure 4. In this figure, the prior distributions used to build the output distributions on Y1 and Y2 are uniform with the upper and lower bounds selected to be wide enough as to ensure that the correct solution lies well with the interval (typically on the order of 20 times the manufacturing standard deviation of the parameter). This can be used in conjunction with a system model to create distributions on the output parameters. These output distributions will in general not be uniform, but will instead tend toward a normal distribution, especially as the number of input parameters increases. This phenomenon is a direct result of the law of large numbers as is well-known in probability and statistics.

We know that the observed values of the output parameters will in general not be exactly the true value but will instead have an additional random measurement error. It is therefore necessary to augment the system model with additional terms to account for the measurement error. This can be as simple as a random measurement error added to the

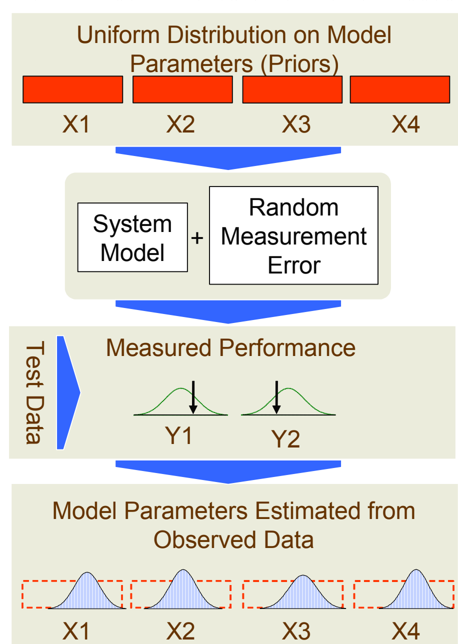

Figure 4: "Maximum Entropy" Inference Based on Uniform Prior Distributions.

output of the system model. The standard deviation of the error term is determined by the known accuracy of the Y-measurement process. The observed values of the output parameters can then be used to extract that portion of the X-distributions that corresponds to input parameter combinations that could have given rise to the observations. Thus, *the original (unbiased) uniform distributions are transformed into biased distributions based only on the observed values of the matching parameters*. The estimated parameters from the maximum entropy analysis can then be used for exact Bayesian Updating of current parameter estimates.

The use of "maximum entropy" priors for Inferenceing does have some repercussions regarding the solution. If the posterior distribution of X's inferred from the observed Y's bumps against the upper or lower limit of the uniform prior interval (e.g. "X1" in Figure 4), this will introduce bias into the solution because the posterior will be truncated. When this happens, the interval of the uniform distribution must be adjusted and the Inference process re-run.

A second repercussion of using uniform distributions is that there is an inherent tradeoff between solution confidence and analysis power. The wider the upper and lower bounds are set on the uniform distributions, the greater the opportunity for the observed data to influence the parameter distributions. On the other hand, as the interval widths are increased, the confidence interval on the inferred X's becomes increasingly large. Thus, the demand for a tight confidence interval in the solution tends to be directly contradictory with the demand for an accurate estimate of parameter means based on the minimum amount of test data. Selection of uniform distribution width is thus a compromise between accuracy and precision.

It should be clear based on the previous discussion how the concepts of Bayesian Updating and Inference allow one to learn something about and update the original input distributions (the X's) based on information collected after-the-fact. This can be done either by *updating* priors using direct observations of the parameters (the X's) or indirectly by updating on distributions *inferred* from a measured output variable (the Y's) translated back into equivalent input distributions. The strengths of this approach are: 1) it is inherently a probabilistic formulation that takes advantage of all available information regarding nominal and confidence estimates of problem parameters; 2) matched status parameters are expressed in terms of probability distributions from which confidence bounds can be estimated; 3) it facilitates continuous updating of model parameters as new data becomes available; 4) the calculation procedure is mathematically rigorous; and 5) the basis of the method is relatively simple and intuitive to understand.

Bayesian Approach to Status Matching

Let us return to the status matching problem articulated in the introduction of this paper and examine it from a Bayesian perspective. If we view status deck matching parameters as being the prior distributions that are to be updated using additional test data, it follows that Bayesian Updating can be a very useful tool for matching status decks to observed test data. In the context of Figure 1, one can imagine that the starting point will be some nominal estimate of what the model parameters are based on past experience and current analysis. As test experience is accrued, each of these new data points could then be used to update beliefs about the distributions that best describe the model parameters.

The basic approach for employing Bayesian methods for status matching is shown in Figure 5. Starting at the upper right corner of the figure, it is presumed that we are given prior distributions on model parameters based on past experience. These distributions on model parameters imply that expected system performance is also a distribution of possible outcomes. These output distributions are easily determined through Monte Carlo simulation methods using the cycle model plus a model for variation of observed performance due to measurement uncertainty. Given an observed value of system performance (as from a test), we can use Inference to infer the parameter update distributions (infer the X's given the Y's). These update distributions are then used with the prior distributions and the update equations to produce posterior distributions on the parameters (the updated distributions, upper left). These posteriors then become the priors for the next test data point that comes along.

Selection of Prior Distributions

If a Bayesian approach to status matching is to be truly useful, it must be repeatable and must also have a reasonable basis in physics and/or mathematics (i.e. it must be defensible when subjected to scrutiny). Selection of the mean of prior distributions used in the update process determines how far the solution is from the correct solution while standard deviation on prior distributions effectively determines how difficult it will

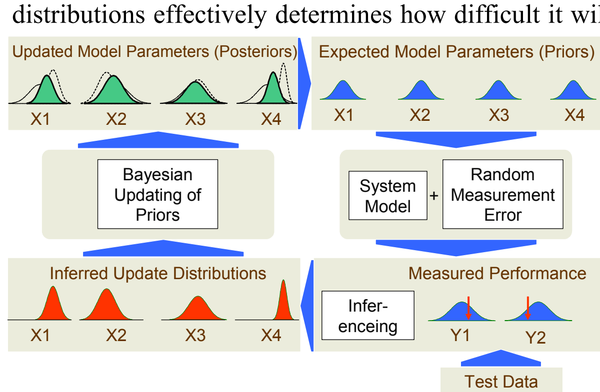

Figure 5: Bayesian View of Status Matching.

be for successive data points to move the solution away from the nominal starting value. The prior distributions thus embody all prior knowledge of the parameters and also influence how much the new data will be able to impact the updated solution.

In many cases (status matching included), the priors will also represent prior *beliefs* as much as prior knowledge. Thus, while Bayesian Updating itself is a mathematically rigorous and exact process, selection of priors is not. Selection of priors can have a big impact on the final solution, and the subjectivity inherent in the selection process is the Achilles' heel of the method.

Some guidance in how to best select priors is provided by realizing that the ultimate objective in status matching is to derive the greatest possible amount of information about the underlying parameter distributions based on the observed performance results. These results will typically take one of two forms: direct observations of model parameters used in the matching process (the X's) or direct observations of the performance outputs (the Y's) from which we must infer something about the settings of the model parameters that gave rise to that output.

The former case where we have direct information regarding the model parameters is typical of the kind of data obtained from component and rig tests. This kind of information is also sometimes available from engine tests when a full suite of internal instrumentation data is available to enable direct observation of model parameters. In this case, the prior distributions on model parameters can be updated using the exact Bayesian Updating of model parameters. The priors are provided by our past history of model parameters (based on past experience and test data), the means of the update data are given by the measured parameter data, and the standard deviation of the update data is given by the expected measurement error of the instrumentation.

In the case where we are given performance output data and (the Y's) are required to update the model parameters (the X's), we must resort to using the Inference techniques illustrated in Figure 3 in order to get the X's with which to update the X-distributions. In this case, the width of the uniform distributions used for Inferenceing will impact the standard deviation of the update distributions on X's. The selection of the upper and lower bounds in this case is dependent on the need for accuracy versus precision.

Evolution of Model Parameter Estimates

An appealing attribute of a Bayesian approach to status matching (aside from its probabilistic treatment of the problem) is the fact that as each incremental data point is collected over time, the Bayesian Updating process provides a ready-made means for tracking the migration of model parameters and confidence bounds over time, as shown in Figure 6. This figure notionally depicts the nominal estimates for a model parameter, X, over time and across product variations. The nominal values and confidence bounds of the parameter are depicted as error bars which change over time as the estimates for the model parameter evolve. The typical scenario envisioned here starts with a nominal estimate of parameters accompanied by a relatively wide spread in the confidence bounds. As pre-production compliance testing takes place, the mean estimates for the parameters change slightly while the confidence bounds shrink considerably. Presumably, lessons from the compliance testing are then incorporated into the production design, which may cause a slight shift in the nominal parameters, accompanied by a temporary increase in confidence interval width. As production experience accrues, the intervals again shrink and reach some stable value. Later, a new production block begins, perhaps with slightly modified manufacturing or testing processes which again causes a temporary increase of confidence interval widths, followed by a convergence to a steady state. One can imagine that if a new product variation was introduced from which the initial parameter estimates were based on production of the previous model, there may again be a temporary increase in uncertainty as new model estimates are found. Each step along this parameter time history represents a Bayesian Update operation.

Application to a Simplified Matching Problem

The focus up to this point has been on development of the abstract theoretical ideas regarding how one could apply Bayesian methods to cycle status matching. However, the best way to gain understanding of the

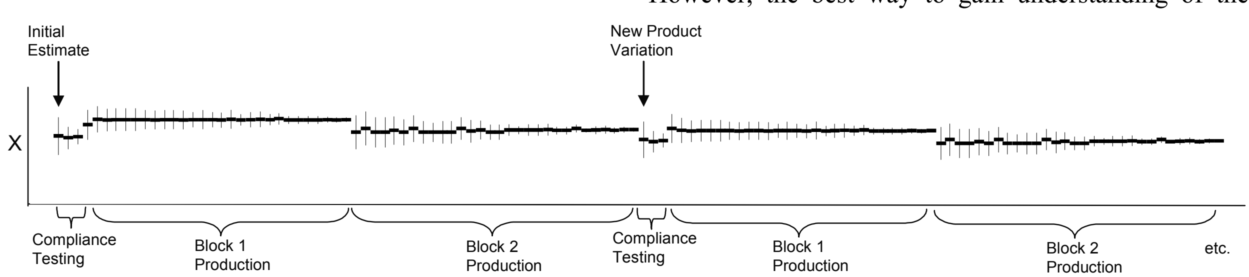

Figure 6: Tracking of Model Parameters and Confidence Bounds Over Time and Between Product Variations.

concepts presented herein is through application. In the interest of presenting the clearest possible view of Bayesian matching concepts without becoming ensnared in the many details attendant to a full-blown cycle status match, this paper uses a simplified surrogate matching problem to explain the basic concepts. This problem consists of a cantilever beam having four parameters that are to be matched to test data: beam length, L; width, b; height, h; and modulus of elasticity, E. These parameters are directly analogous to the matching parameters available in a cycle deck (flow and efficiency scalars, etc.). In addition to the four model parameters, the beam has two user-controlled usage parameters: point load on the beam, P, and point of load application, a. These are directly analogous to the flight conditions and throttle setting in a cycle deck. Finally, this problem has two performance parameters with which to match performance: maximum stress, Smax, and tip deflection, Ymax. These parameters are directly analogous to the cycle deck performance parameters (thrust, SFC, etc.).

Each of these parameters (L, b, h, E) has an initial nominal value which is estimated from previous experience, engineering judgment, historical data, etc. In addition, the beam parameters have a prescribed estimate of manufacturing variation. This variation in conjunction with the nominal parameter estimate forms the prior distributions on L, b, h, and E that are the starting point for the Bayesian Updating.

Revised estimates of the parameter distributions are obtained through matching of test data to model results. The matching data consists of 22 load cases. Each of these cases has a measured value of the load conditions (P and a), and a corresponding measurement uncertainty in P and a. Further, each load case results in a pair of measured performance parameters (Smax and Ymax). The measurement Smax, and Ymax is not perfect but is presumed to have some measurement error prescribed by the accuracy of the measurement process. Finally, we should presume that matching data is available not only in the form of measured performance (Smax and Ymax), but is sometimes also available as direct measurements of the L, b, h, and E parameters themselves. This same situation also arises in cycle matching: some test points only measure operating conditions and performance outputs while some data points will be heavily instrumented rigs or test engines having sufficient sensor inputs to directly measure some of the parameters of interest.

The objective of such a cantilever beam matching problem is to find the values of L, b, h, and E that yield the best possible agreement between model predictions and test data. This must occur while simultaneously providing insight regarding model accuracy and confidence bounds. It is desirable that the method should utilize all information available (deterministic and probabilistic) and should also be capable of accommodating the wide variety of test data that may be available.

The Bayesian-based method proposed to address these needs is illustrated in Figure 7. Starting in the center, it can be presumed that nominal values for the beam parameters are given in the initial problem

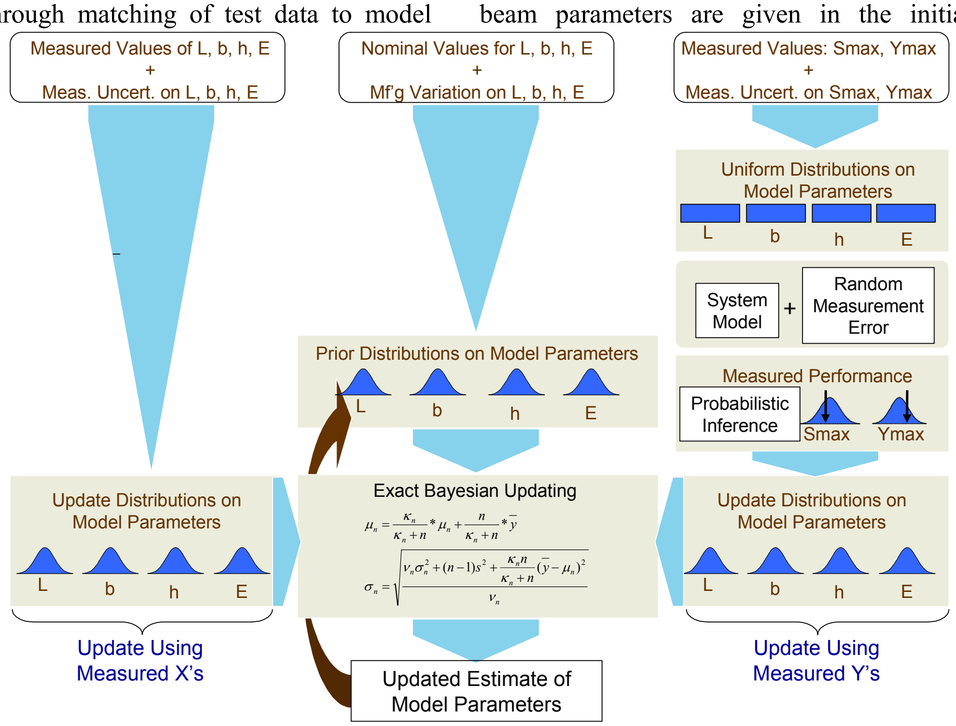

Figure 7: Bayesian Updating Method for Cantilever Beam Surrogate Problem.

specification. These can be used in conjunction with estimates on the expected manufacturing variation to yield starting prior distributions on L, b, h, and E. These distributions can be combined with distributions on L, b, h, and E estimated from matching data to yield updated distributions on the parameters (posteriors). If normal distributions are assumed, the exact Bayesian Updating technique described previously can be used to do this. Finally, the updated distributions on the model parameters can be used as the starting priors for the next matching data point that becomes available. This enables one to obtain a running estimate of each model parameter as additional data becomes available.

The test data used in the Bayesian Updating of model parameters will typically take one of two forms. Either direct measurements of model parameters (the X's) will be available for use in updating, or measurements of performance parameters (the Y's) must be used to infer underlying model parameters for updating. In the former case, the Updating process is simple: the measured values of L, b, h, and E provide the mean estimate of the parameter while the measurement uncertainty on L, b, h, and E provides an estimate on standard deviation. This case is shown on the left side of Figure 7. The matching data can be used directly in the Bayesian Updating on model parameters.

In the latter case where only measured performance data is available, some additional analysis is necessary to infer parameter distributions corresponding to the observed performance. This can be accomplished using the "maximum entropy" updating technique described earlier where upper and lower bounds are selected for each parameter and used to create a uniform distribution on all model parameters. Monte Carlo simulation is then used to build a distribution on performance parameters. Measurement uncertainty on usage parameters (P and a) can be accounted for by applying a distribution to these parameters based on the measurement uncertainty of P and a during test. Additionally, measurement uncertainty on the performance parameters (Smax and Ymax) can be accounted for by applying an additional error term on the system model calculations. This error should have a normal distribution with a standard deviation given by the measurement accuracy of the performance parameter on test. The resultant distributions on performance parameters can then be compared to observed performance data and used to infer distributions on L, b, h, and E using the techniques described previously. These inferred distributions are then used in the parameter updating on L, b, h, and E just as before.

Typical Results

Typical results from Bayesian Updating of the beam problem for 22 data points are shown in Figure 8 and Figure 9. Figure 8 shows the prior distributions on model parameters that was the starting point for the analysis (dashed lines). The solid lines show the final distribution after Bayesian Updating using 22 data points and assuming upper and lower bounds of ±50σ of the original prior distributions. The actual parameter values are L=22.15 in, b=1.092 in, h=2.08 in, and E=9.995E6 psi. Thus, all four parameters move closer to the correct solution than when they started.

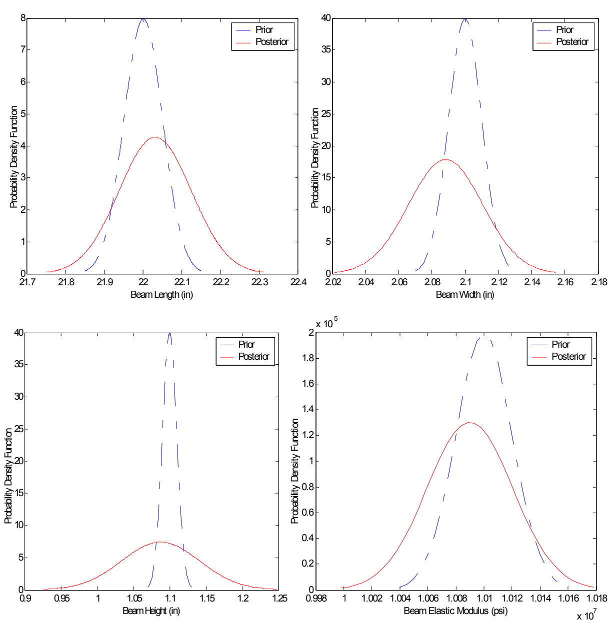

Figure 8: Prior (Dashed) and Posterior (Solid) Distributions on Beam Parameters 22 Update Points.

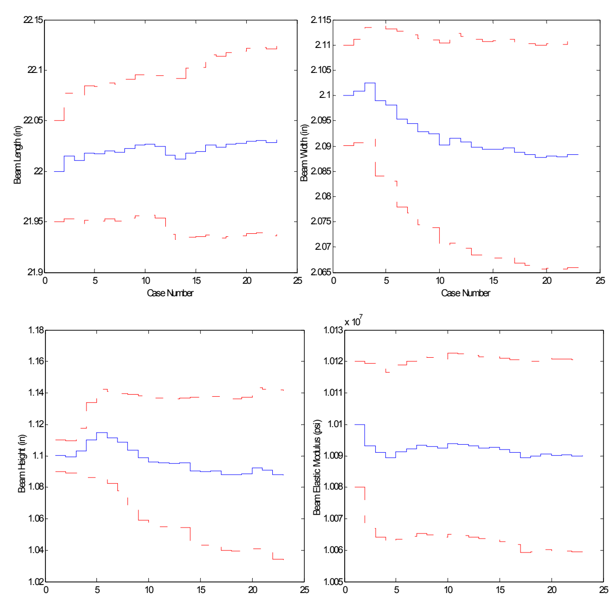

Figure 9: Evolution of Model Parameter Estimates and ±1σ Confidence Bounds as Each Data Point Is Added.

However, the estimate of standard deviation on the beam parameters is considerably larger than the initial estimate due to the wide range of bounds selected for the uniform distributions. If $\pm 20\sigma$ had been selected, the confidence bounds would have been narrower, but the mean estimate of the parameters would have shifted less. This illustrates the basic tradeoff necessary when employing this method: demands for tight confidence intervals necessarily result in less ability to move the estimate of mean parameter values based on new data. Conversely, test data can be used to narrow the confidence bands on parameters, but at the expense of ability to effect a change in the parameter means.

Figure 9 shows the evolution of model parameter estimates (solid line) and $\pm 1\sigma$ confidence intervals (dashed) for L, b, h, and E model parameters. As was shown previously, all four parameters move closer to their correct values based on the test data provided. Presumably, the nominal estimates would continue to move further toward the nominal estimate as further data is obtained. Similarly, note how the $\pm 1\sigma$ confidence intervals increase from their initial values and stabilize on a higher value. This is an artifact of how the upper and lower bounds on the uniform distributions are selected. Narrower bounds would have led to a smaller confidence interval but would also have resulted in a smaller shift in the mean of the distributions. In the limit, as the upper and lower bounds on the uniform distributions approach zero, one would recover the original nominal distributions no matter how many data points were available for use in the matching process.

Conclusions

This paper has attempted to introduce the fundamental concepts of Bayesian Updating and Inference and illustrate how they can be applied to the engine cycle model matching process. The method presented here is only one (very simple) implementation among many possibilities, and is by no means a whole and refined cycle matching method. Nevertheless, the basic approach shows considerable promise in that it is inherently probabilistic, reasonably intuitive, and can use a wide variety of data to provide a running estimate of model parameters. The beam problem example illustrated the strengths and weaknesses of the current implementation. These are the ability to incrementally update parameter estimates using probabilistic data, and the inevitable tradeoff between solution accuracy and precision, respectively. This paper will hopefully spark further interest in applying Bayesian concepts to status matching, the development of which could lead to substantial payoffs in terms of cycle model accuracy and confidence bounds.

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